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A NOTE ON μ-RESOLVABLE BIB DESIGNS

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SUMMARY

In this paper a method of construction of a class of μ-resolvable balanced incomplete block designs has been presented. All the designs constructed by this method under certain restrictions are tabulated.

Keywords : Mutually Orthogonal Latin squares; Balanced incomplete block designs.

Introduction

A balanced incomplete block (BIB) design with parameters v, b. r, k and λ is called μ-resolvable whenever the blocks can be separated into t groups of m blocks each such that each group contains every treatment exactly μ times. These designs were initially introduced by Bose [1] for $\mu = 1$. The construction methods and combinatorial properties of μ-resolvable BIB designs have been widely discussed in literature, e.g., Kageyama, [2] [3], Mohan [5], Shrikhande and Raghavarao [6]. Recently Subramani [7] gave a method for constructing a series of BIB designs by using a set of mutually orthogonal Latin squares (MOLS). In this paper it is proved that the designs constructed by Subramani [7] are also μ-resolvable. The designs obtained are given in Table 1. These designs are different from those listed by Kageyama and Mohan [4].

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2. Main Results

Subramani [7] gave the following method of construction of a series of BIB designs.

THEOREM 2.1. *The existence of s mutually orthogonal Latin squares of side n implies the existence of BIBD with parameters*

$$v = n,\ b = n(n-1),\ r = p(n-1),\ k = p \text{ and } \lambda = p(p-1), \quad (2.1)$$

where $2 \leq p \leq s$

The method of the construction is as follows :

If we superimpose p $(2 \leqslant p \leqslant s)$ mutually orthogonal Latin squares of side n, we get the pth order hyper-graeco Latin squares of side n. By forming each cell as a block and omitting the blocks in which the elements are equal we obtain the BIBD with the parameters given in (2.1).

THEOREM 2.2. *The BIB designs obtained in Theorem 2.1 are μ-resolvable.*

Proof: Without loss of generality we can assume that all MOLS are in semi-standard form, that is, the elements in the first row for all Latin squares are in natural order. Then omitting the first row in the hyper-graeco Latin square and forming the elements in the remaining cells as blocks, the resulting BIB design is a μ-resolvable, where each row forms a group in which each element appears μ-times (which is true by the definition of Latin squares).

Further if $t = t_1 t_2$, then we can show that the above μ-resolvable BIBD is also $t_1\mu$-resolvable and $t_2\mu$-resolvable. For example we know that the BIBD with parameters $v = 7, b = 42, r = 18, k = 3, \lambda = 6, \mu = 3, m = 7$, and $t = 6$ is 3-resolvable, obtained by taking the cells contained in each row of the HGLS of order 3 as a group. Similarly by taking every two (three) rows as a group, we can show that the resulting BIB design is 6 (9)-resolvable.

A list of available μ-resolvable designs is given in Kageyama and Mohan [4] for $v \leqslant 125, b \leqslant 250, \lambda \leqslant 100$ and $3 \leqslant k \leqslant v - 3$ with $\mu > 2$. In Table 1 we give all designs constructed by the method discussed above for $v \leqslant 20, b < 350, \lambda \leqslant 250$ and $3 < k \leq v - 3$ with $\mu \geqq 3$.

The reference number with asterisk (*) is also listed in Kageyama and Mohan [4].

TABLE 1

Sl. No.	*v*	*b*	*r*	*k*	λ	μ	*m*	*t*
1	7	42	18	3	6	3	7	6
2	7	42	18	3	6	6	14	3
3	7	42	18	3	6	9	21	2
4	7	42	24	4	12	4	7	6
5	7	42	24	4	12	8	14	3
6	7	42	24	4	12	12	21	2
7	8	56	21	3	6	3	8	7
8	8	56	28	4	12	4	8	7
9	8	56	35	5	20	5	8	7
10	9	72	24	3	6	3	9	8
11	9	72	24	3	6	6	18	4
12	9	72	24	3	6	12	36	2
13	9	72	32	4	12	4	9	8
14	9	72	32	4	12	8	18	4
15	9	72	32	4	12	16	36	2
16	9	72	40	5	20	5	9	8
*17	9	72	40	5	20	10	18	4
*18	9	72	40	5	20	20	36	2
19	9	72	48	6	30	6	9	8
20	9	72	48	6	30	12	18	4
21	9	72	48	6	30	24	36	2
22	11	110	30	3	6	3	11	10
23	11	110	30	3	6	6	22	5
24	11	110	30	3	6	15	55	2
25	11	110	40	4	12	4	11	10
26	11	110	40	4	12	8	22	5
27	11	110	40	4	12	20	55	2
28	11	110	50	5	20	5	11	10
29	11	110	50	5	20	10	22	5
30	11	110	50	5	20	25	55	2
31	11	110	60	6	30	6	11	10
32	11	110	60	6	30	12	22	5
33	11	110	60	6	30	30	55	2
34	11	110	70	7	42	7	11	10
35	11	110	70	7	42	14	22	5
36	11	110	70	7	42	35	55	2
37	11	110	80	8	56	8	11	10
38	11	110	80	8	56	16	22	5
39	11	110	80	8	56	40	55	2
40	13	156	36	3	6	3	13	12
41	13	156	36	3	6	6	26	6
42	13	156	36	3	6	9	39	4
43	13	152	36	3	6	12	52	3
44	13	156	36	3	6	18	78	2

Sl. No.	*v*	*b*	*r*	*k*	λ	μ	*m*	*t*
45	13	156	48	4	12	4	13	12
46	13	156	48	4	12	8	26	6
47	13	156	48	4	12	12	39	4
48	13	156	48	4	12	16	52	3
49	13	156	48	4	12	24	78	2
50	13	156	60	5	20	5	13	12
51	13	156	60	5	20	10	26	6
52	13	156	60	5	20	15	39	4
53	13	156	60	5	20	20	52	3
54	13	156	60	5	20	20	78	2
55	13	156	72	6	30	6	13	12
56	13	156	72	6	30	12	26	6
57	13	156	72	6	30	18	39	4
58	13	156	72	6	30	24	52	3
59	13	156	72	6	30	36	78	2
60	13	156	84	7	42	7	13	12
61	13	156	84	7	42	14	26	6
62	13	156	84	7	42	21	39	4
63	13	156	84	7	42	28	52	3
64	13	156	84	7	42	42	78	2
65	13	156	96	8	56	8	13	12
66	13	156	96	8	56	16	26	6
67	13	156	96	8	56	24	39	4
68	13	156	96	8	56	32	52	3
69	13	156	96	8	56	48	78	2
70	13	156	108	9	72	9	13	12
71	13	156	108	9	72	18	26	6
72	13	156	108	9	72	27	39	4
73	13	156	108	9	72	36	52	3
74	13	156	108	9	72	54	78	2
75	13	156	120	10	90	10	13	12
76	13	156	120	10	90	20	26	6
77	13	156	120	10	90	30	39	4
78	13	156	120	10	90	40	52	3
79	13	156	120	10	90	60	78	2
80	16	240	45	3	6	3	16	15
81	16	240	45	3	6	9	48	5
82	16	240	45	3	6	15	80	3
83	19	240	60	4	12	4	16	15
84	16	240	60	4	12	12	48	5
85	16	240	60	4	12	20	80	3
86	16	240	75	5	20	5	16	15
87	16	240	75	5	20	15	48	5
88	16	240	75	5	20	25	80	3
89	16	240	90	6	30	6	16	15
90	16	240	90	6	30	18	48	5

Sl. No.	v	b	r	k	λ	μ	m	t
91	16	240	90	6	30	30	80	3
*92	16	240	105	7	42	7	16	15
*93	16	240	105	7	42	21	48	5
*94	16	240	105	7	42	35	80	3
95	16	240	120	8	56	8	16	15
96	16	240	120	8	56	24	48	5
97	16	240	120	8	56	40	80	3
98	16	240	135	9	72	9	16	15
99	16	240	135	9	72	27	48	5
100	16	240	135	9	72	45	80	3
101	16	240	150	10	90	10	16	15
102	16	240	150	10	90	30	48	5
103	16	240	150	10	90	50	80	3
104	16	240	165	11	110	11	16	15
105	16	240	165	11	110	33	48	5
106	16	240	165	11	110	55	80	3
107	16	240	180	12	132	12	16	15
108	16	240	180	12	132	36	48	5
109	16	240	180	12	132	60	80	3
110	16	240	195	13	156	13	16	15
111	16	240	195	13	156	39	48	5
112	16	240	195	13	156	65	80	3
113	17	272	48	3	6	3	17	16
114	17	272	48	3	6	6	34	8
115	17	372	48	3	6	12	68	4
116	17	272	48	3	6	24	136	2
117	27	272	64	4	12	4	17	16
118	17	272	64	4	12	8	34	8
119	17	272	64	4	12	16	68	4
120	17	272	64	4	12	32	131	2
121	17	272	80	5	20	5	17	16
122	17	272	80	5	20	10	34	8
123	17	272	80	5	20	20	68	4
124	17	272	80	5	20	40	136	2
125	17	272	96	6	30	6	17	16
126	17	272	96	6	30	12	34	8
127	17	272	96	6	30	24	68	4
128	17	272	96	6	30	48	136	2
129	17	272	112	7	42	7	17	16
130	17	272	112	7	42	14	34	8
131	17	272	112	7	42	28	68	4
132	17	272	112	7	42	56	136	2
133	17	272	128	8	56	8	17	16
134	17	272	128	8	56	16	34	8
15	17	272	128	8	56	32	68	4
136	17	272	128	8	56	64	136	2
137	17	272	144	9	72	9	17	16

Sl. No.	v	b	r	k	λ	μ	m	t
138	17	272	144	9	72	18	34	8
139	17	272	144	9	72	36	68	4
140	17	272	144	9	72	72	136	2
141	17	272	160	10	90	10	17	16
142	17	272	160	10	90	20	34	8
143	17	272	160	10	90	40	68	4
144	17	272	160	10	90	80	136	2
145	17	272	176	11	110	11	17	16
146	17	272	176	11	110	22	34	8
147	17	272	176	11	110	44	68	4
148	17	272	176	11	110	88	136	2
149	17	272	192	12	132	12	17	16
150	17	272	192	12	132	24	34	8
151	17	272	192	12	132	48	68	4
152	17	272	192	12	132	96	136	2
153	17	272	208	13	156	13	17	16
154	17	272	208	13	156	26	34	8
155	17	272	208	13	156	52	68	4
156	17	272	208	13	156	104	136	2
157	17	272	224	14	182	14	17	16
158	17	272	224	14	182	28	34	8
159	17	272	224	14	182	56	68	4
160	17	272	224	14	182	112	136	2
161	19	342	54	3	6	3	19	18
162	19	342	54	3	6	6	38	9
163	19	342	54	3	6	9	57	6
164	19	342	54	3	6	18	114	3
165	19	342	54	3	6	27	171	2
166	19	342	72	4	12	4	19	18
167	19	342	72	4	12	8	38	9
168	19	342	72	4	12	12	57	6
169	19	342	72	4	12	24	114	3
170	19	342	72	4	12	36	171	2
171	19	342	90	5	20	5	19	18
172	19	342	90	5	20	10	38	9
173	19	342	90	5	20	15	57	6
174	19	342	90	5	20	30	114	3
175	19	342	90	5	20	45	171	2
176	19	342	108	6	30	6	19	18
177	19	342	108	6	30	12	38	9
178	19	342	108	6	30	18	57	6
179	19	342	108	6	30	36	114	3
180	19	342	108	6	30	54	171	2
181	19	342	126	7	42	7	19	18
182	19	342	126	7	42	14	38	9
183	19	342	126	7	42	21	57	6
184	19	342	126	7	42	42	114	3

Sl. No.	v	b	r	k	λ	μ	m	t
185	19	342	126	7	42	63	171	2
186	19	342	144	8	56	8	19	18
187	19	342	144	8	56	16	38	9
188	19	342	144	8	56	24	57	6
189	19	342	144	8	56	48	114	3
190	19	342	144	8	56	72	171	2
191	19	342	162	9	72	9	19	18
192	19	342	162	9	72	18	38	9
193	19	342	162	9	72	27	57	6
194	19	342	162	9	72	54	114	3
195	19	342	162	9	72	81	171	2
196	19	342	180	10	90	10	19	18
197	19	342	180	10	90	20	38	9
198	19	342	180	10	90	30	57	6
199	19	342	180	10	90	60	114	3
200	19	342	180	10	90	90	171	2
201	19	342	198	11	110	11	19	18
202	19	342	198	11	110	22	38	9
203	19	342	198	11	110	33	57	6
204	19	342	198	11	110	66	114	3
205	19	342	198	11	110	99	171	2
206	19	342	216	12	132	12	19	18
207	19	342	216	12	132	24	38	9
208	19	342	216	12	132	36	57	6
209	19	342	216	12	132	72	114	3
210	19	342	216	12	132	108	171	2
211	19	342	234	13	156	13	19	18
212	19	342	234	13	156	26	38	9
213	19	342	234	13	156	39	57	6
214	19	342	234	13	156	78	114	3
215	19	342	234	13	156	117	171	2
216	19	342	252	14	182	14	19	18
217	19	342	252	14	182	28	38	9
218	19	342	252	14	182	42	57	6
219	19	342	252	14	182	84	114	3
220	19	342	252	14	182	126	171	2
221	19	342	270	15	210	15	19	18
222	19	342	270	15	210	30	38	9
223	19	342	270	15	210	45	57	6
224	19	342	270	15	210	90	114	3
225	19	342	270	15	210	135	171	2
226	19	342	288	16	240	16	19	18
227	19	342	288	16	240	32	38	9
228	19	342	288	16	240	48	57	6
229	19	342	288	16	240	96	114	3
230	19	342	288	16	240	144	171	2

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REFERENCES

[1] Bose, R. C. (1942) : A note on the resolvability of balanced block designs. *Sankhya* 6 : 105-110.

[2] Kageyama, S. (1972) : A survey of resolvable solutions of balanced incomplete block designs. *Int. Stat. Rev.* 40 ; 269-273.

[3] Kageyama, S. (1973) : On μ-resolvable and affine μ-resolvable balanced incomplete block designs. *Ann. Statist.*, 1 : 195-203.

[4] Kageyama, S. and Mohan, R. N. (1983) : On μ-resolvable BIB designs. *Discrete Math.*, 45 : 113-122.

[5] Mohan, R. N. (1980) : A note on the construction of certain BIB designs. *Discrete Math.*, 29 : 209-211.

[6] Shrikhande, S. S. and Raghavarao, D. (1963) : A method of construction of incomplete block designs. *Sankhya A*, 25 : 399-402.

[7] Subramani, J. (1990) : A method of construction of balanced incomplete block designs. *AMSE Review*, 15 (4) : 27-29.